

KGŠM

Ako súvisia matice s komprimovaním obrázkov

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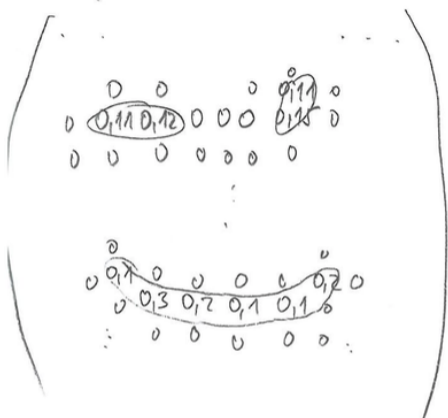
Čo sú to matice

Matica = tabuľka čísel

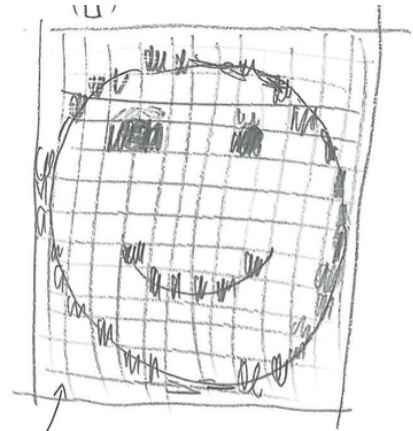


157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	5	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	85	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	90	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

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155	182	163	74	75	62	33	17	110	210	180	154
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206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	85	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	90	2	109	249	215
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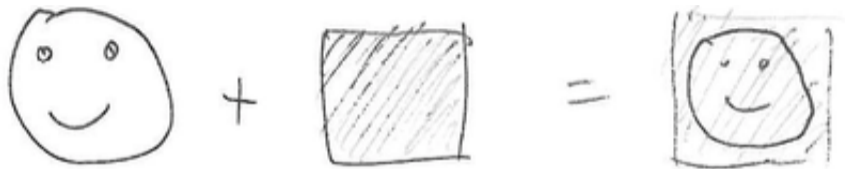
← 0,83
 ← 0,12

Matice môžu byť

veľké, malé, úzke, široké

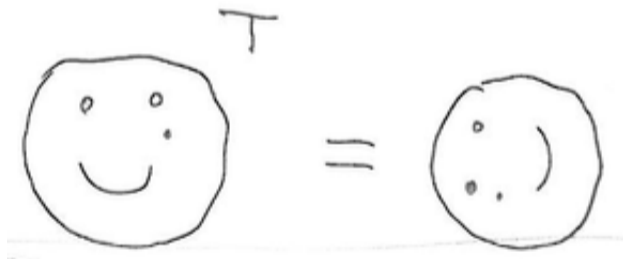
Sčítanie matíc

Rovnako veľké matice môžeme sčítať alebo odčítať.



Prevrátenie matice

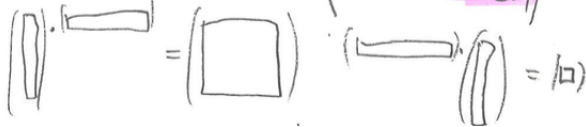
Matice môžeme prevrátiť (transponovať).



Násobenie matíc

Niektoré matice môžeme medzi sebou násobiť.

$$\begin{pmatrix} 4 & 2 \\ 1 & 0 \\ 7 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 \cdot 1 + 2 \cdot 0 & 4 \cdot 2 + 2 \cdot 1 \\ \cancel{1 \cdot 1 + 0 \cdot 0} & 1 \cdot 2 + 0 \cdot 1 \\ 7 \cdot 1 + 3 \cdot 0 & 7 \cdot 2 + 3 \cdot 1 \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 1 & 2 \\ 7 & 15 \end{pmatrix}$$



Násobenie matíc

Násobenie matíc všeličo znamená.

$$\begin{matrix} \circ & \circ & \cdot & \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & & 1 & 1 & \\ \vdots & & & & \\ 1 & \dots & & 0 & 0 \end{pmatrix} & = & \begin{matrix} \circ & \circ & \cdot & \circ & \circ \\ \circ & \circ & \cdot & \circ & \circ \\ \vdots & \vdots & \cdot & \vdots & \vdots \\ \circ & \circ & \cdot & \circ & \circ \end{matrix} \end{matrix}$$

$$\begin{matrix} \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & & 1 & \\ \vdots & & & \\ 1 & \dots & & 0 & 0 \end{pmatrix} \cdot \begin{matrix} \circ & \circ & \cdot & \circ & \circ \\ \circ & \circ & \cdot & \circ & \circ \\ \vdots & \vdots & \cdot & \vdots & \vdots \\ \circ & \circ & \cdot & \circ & \circ \end{matrix} & = & \begin{matrix} \circ & \circ & \cdot & \circ & \circ \\ \circ & \circ & \cdot & \circ & \circ \\ \vdots & \vdots & \cdot & \vdots & \vdots \\ \circ & \circ & \cdot & \circ & \circ \end{matrix} \end{matrix}$$

Maticové operácie

Maticové operácie môžeme skladať.

$$\left(\begin{array}{c} \text{Smiley} \\ \text{Identity Matrix} \end{array} \right)^T = \text{Smiley}$$

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Maticové operácie

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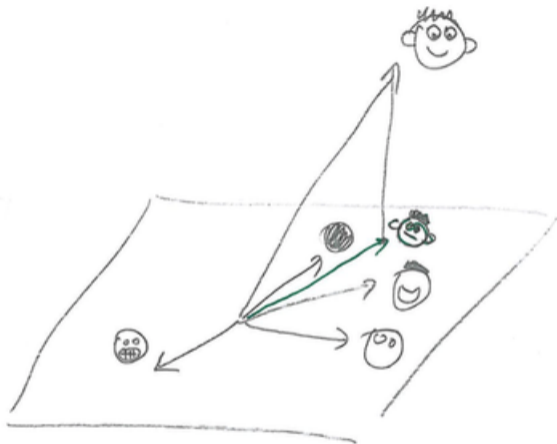
$$\text{Smiley}^T \cdot \begin{pmatrix} 0 & 0 & & 1 \\ 0 & & \dots & \\ 1 & & & \\ & & 0 & 0 \end{pmatrix} = \text{Smiley}$$

$$\left(\begin{pmatrix} 0 & 0 & & 1 \\ 0 & & \dots & \\ 1 & & & \\ & & 0 & 0 \end{pmatrix} \cdot \text{Smiley} \right)^T = \text{Smiley}$$

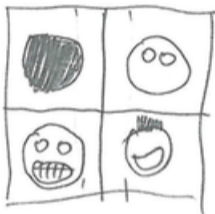
Matice môžeme merať.

Medzi maticami vieme merať vzdialenosti





$$\begin{array}{c} \text{neutral face} \\ \text{smiling face} \end{array} = B_0 + B_1 \cdot \text{shaded circle} + B_2 \cdot \text{sad face} + B_3 \cdot \text{grumpy face} + B_4 \cdot \text{neutral face} + B_5 \cdot \text{smiling face}$$



← základné ksidhtíky

Tváre

Sample of original faces before running PCA:



Obr.: Zdroj: <https://github.com/gbuesing/pca/tree/master/examples>

Rekonštrukcia pomocou 36 hlavných komponentov



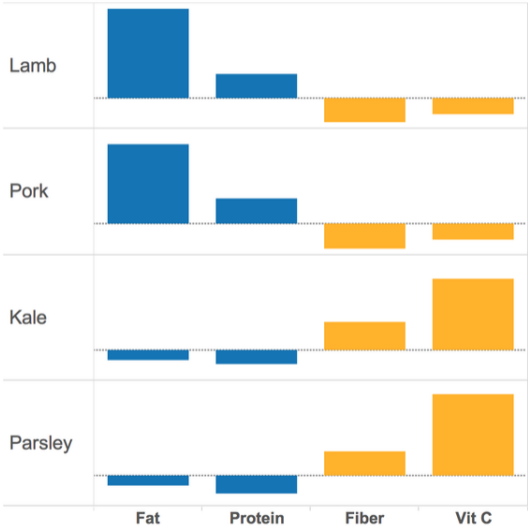
Obr.: Zdroj: <https://github.com/gbuesing/pca/tree/master/examples>

'Eigenfaces'



Obr.: Zdroj: <https://github.com/gbuesing/pca/tree/master/examples>

Iný príklad



Obr.: Zdroj:

Iný príklad



Obr.: Zdroj:

<https://www.quora.com/What-is-an-intuitive-explanation-for-PCA>

Guinea Hen



Obr.: Zdroj: wiki

Ale teraz seriózne

$P = 10304$

$n = 400$

X

$S_{v_i} = X^T X v_i = \lambda_i v_i$ $\leftarrow p \times 1$

$XX^T u_i = \lambda_i u_i$ $\leftarrow n \times 1$ easy to get

$X^T X [X^T u_i] = \lambda_i [X^T u_i]$

faces_pca\$rot


400

10304

eigenfaces

$\text{face} = \beta_0 + \beta_1 \text{eigenface}_1 + \beta_2 \text{eigenface}_2 + \beta_3 \text{eigenface}_3 + \beta_4 \text{eigenface}_4 + \epsilon$

Ďakujem za pozornosť.

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-  [KMFPVUMB](#)